

Boundary Layer Flow & Heat Transfer of an Unsteady Dusty Fluid over a Stretching Sheet

S.K.Mishra and A.K.Rauta

Abstract- The unsteady flow and heat transfer of a viscous incompressible dusty fluid over a vertical stretching sheet is studied. The governing partial differential equations are solved by reducing into ordinary differential equations using similarity transformations and the solutions have been found using well known Runge-Kutta method with help of Shooting technique. The effect of pertinent flow parameters such as Unsteady parameter, Froud number, Grashof number, Prandtl number, Eckert number, Volume fraction, fluid interaction parameter etc on heat transfer are investigated with help of tables and graphs. We have found that our result is good agreement with previously published results. It is found that the thermal and momentum boundary layer thickness decreases on the increase of unsteady parameter and the temperature of both fluid phase as well as particle phase are enhancing on the increase of Eckert number. It is also noticed that the rate of cooling is faster for higher prandtl number and unsteady parameter.

AMS classification: 76T10, 76T15

Keywords:

Boundary layer flow, Eckert number, Fluid – particle interaction parameter, Froud number, Grashof number, Prandtl number, Shooting techniques, Stretching sheet, Unsteady parameter, Volume fraction.

1. Introduction:

The unsteady flow and heat transfer of two phase viscous incompressible flow over a stretching sheet is occurring in several industrial applications and technologies. The practical applications are underground disposable of radioactive waste materials, exothermic and endothermic reactions, centrifugal separation of particles, blood rheology, flow through packed beds, sedimentation etc. The momentum and Heat transfer in the laminar boundary layer flow on a moving surface is also important for both practical as well as theoretical point of view because of their wide application in heat removal from nuclear fuel debris, the aerodynamic extrusion of plastic sheet, glass blowing, cooling or drying of papers, drawing plastic films, extrusion of polymer melt-spinning process and heat treated materials traveling on conveyer belt etc. Due to this fact several researchers motivated to study the effect of momentum and heat transfer.

The study of the boundary layer flow over a stretched surface moving with a constant velocity was initiated by Sakiadis B.C.[18] in 1961. Then many researchers extended the above study with the effect of Heat Transfer. Tsou et.al [20] studied the effect of Heat Transfer and experimentally confirmed the numerical result of Sakiadis. Grubka et.al[9] investigated the temperature field in the flow over a stretching surface when subject to uniform heat flux. Sharidan[19] presented similarity solutions for unsteady boundary layer flow and heat Transfer due to stretching sheet. A numerical solution for laminar thermal boundary over a flat plate with convective surface boundary condition was analyzed by Aziz[1]. Chen [6] investigated mixed convection of a power law fluid past a stretching

surface in presence of thermal radiation and magnetic field. Crane [13] has obtained the Exponential solution for planar viscous flow of linear stretching sheet. Chamakha et.al.[7] Investigated the unsteady magneto hydrodynamics boundary layer flow of viscous incompressible electrical conducting fluid along a semi infinite vertical permeable plate. Nandkeolyar et.al. [16] have studied the effect of ramped surface temperature on the flow and heat transfer of a viscous, incompressible and electrically conducting dusty fluid in presence of transverse magnetic field, M Das et.al. [14] recently have studied the unsteady hydro magnetic flow of a Heat absorbing Dusty Fluid past a permeable vertical plate with ramped temperature. B.J. Gireesha et.al[4] have studied the effect of hydrodynamic laminar boundary layer flow and heat Transfer of a dusty fluid over an unsteady stretching surface in presence of non uniform heat source/sink. They have examined the Heat Transfer characteristics for two type of boundary conditions namely variable wall temperature and variable Heat flux. G.K.Ramesh et.al [8] have investigated the momentum and heat transfer characteristics in hydrodynamic flow of dusty fluid over an inclined stretching sheet with non uniform heat source/sink. B.G. Gireesh et.al [3] also studied the mixed convective flow a dusty fluid over a stretching sheet in presence of thermal radiation, space dependent heat source/sink.

In this paper the study of effect of different flow parameters on unsteady boundary layer and heat transfer of a dusty fluid over a stretching sheet have investigated. The problem of two phase suspension flow is solved in the

frame work of a model of a two-way coupling model or a two-fluid approach.

Here, the particles will be allowed to diffuse through the carrier fluid i.e. the random motion of the particles shall be taken into account because of the small size of the particles. This can be done by applying the kinetic theory of gases and hence the motion of the particles across the streamline due to the concentration and pressure diffusion. We have considered the terms related to the heat added to the system to slip-energy flux in the energy equation of particle phase. The momentum equation for particulate phase in normal direction, heat due to conduction and viscous dissipation in the energy equation of the particle phase have been considered for better understanding of the boundary layer characteristics. The effects of volume fraction on skin friction, heat transfer and other boundary layer characteristics also have been studied. The governing equation are reduced into system of ODEs and solved by Shooting Technique using Runge-Kutta Method with help of FORTRAN-77.

2.Mathematical Formulation and Solution: Consider an unsteady two dimensional laminar boundary layer flow of an incompressible viscous dusty fluid over a vertical stretching sheet .The flow is generated by the action of two equal and opposite forces along the x-axis and y-axis being normal to the flow . The sheet being stretched with the

velocity $U_w(x)$ along the x-axis, keeping the origin fixed in the fluid of ambient temperature T_∞ . Both the fluid and the dust particle clouds are suppose to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size and number density of the dust particle is taken as a constant throughout the flow.

The governing equations of unsteady two dimensional boundary layer incompressible flows of dusty fluids are given by

$$\frac{\partial u_f}{\partial t} + \frac{\partial \vec{F}(u_f)}{\partial x} + \frac{\partial \vec{G}(u_f)}{\partial y} + H(u_f) = S(u_f, u_p, T, T_p) \quad (2.1)$$

$$\frac{\partial u_p}{\partial t} + \frac{\partial \vec{F}(u_p)}{\partial x} + \frac{\partial \vec{G}(u_p)}{\partial y} + H(u_p) = S_p(u_f, u_p, T, T_p) \quad (2.2)$$

$$\text{Where } H(u_f) = 0, H(u_p) = 0$$

$$\vec{F}(u_f) = \begin{bmatrix} u \\ (1-\phi)\rho u^2 \\ \rho c_p u T \end{bmatrix}, \vec{F}(u_p) = \begin{bmatrix} \rho_p u_p \\ \rho_p u_p^2 \\ \rho_p c_s u_p T_p \end{bmatrix}$$

$$\vec{G}(u_f) = \begin{bmatrix} v \\ (1-\phi)\rho uv \\ \rho c_p v T \end{bmatrix}, \vec{G}(u_p) = \begin{bmatrix} \rho_p v_p \\ \rho_p u_p v_p \\ \rho_p c_s v_p T_p \end{bmatrix}$$

$$S_p(u_f, u_p, T, T_p) = \begin{bmatrix} 0 \\ \frac{\partial}{\partial y} \left(\phi \mu_s \frac{\partial u_p}{\partial y} \right) + \frac{\rho_p}{\tau_p} (u - u_p) + \phi (\rho_s - \rho) g \\ \frac{\partial}{\partial y} \left(\phi \mu_s \frac{\partial v_p}{\partial y} \right) + \frac{\rho_p}{\tau_p} (v - v_p) \\ \frac{\partial}{\partial y} \left(\phi k_s \frac{\partial T_p}{\partial y} \right) - \frac{\rho_p}{\tau_p} (u - u_p)^2 + \phi \mu_s \left(u_p \frac{\partial^2 u_p}{\partial y^2} + \left(\frac{\partial u_p}{\partial y} \right)^2 \right) - \frac{\rho_p c_s}{\tau_p} (T_p - T) \end{bmatrix}$$

$$S(u_f, u_p, T, T_p) = \begin{bmatrix} 0 \\ \mu \frac{\partial^2 u}{\partial y^2} - \frac{\rho_p}{\tau_p} (u - u_p) + g \beta^* (T - T_\infty) \\ k (1 - \phi) \frac{\partial^2 T}{\partial y^2} + \frac{\rho_p c_s}{\tau_p} (T_p - T) + \frac{\rho_p}{\tau_p} (u_p - u)^2 + \mu (1 - \phi) \left(\frac{\partial u}{\partial y} \right)^2 \end{bmatrix}$$

With boundary conditions

$$\left. \begin{aligned} u = U_w(x) = \frac{cx}{1-at}, v = 0 \text{ at } y = 0 \\ \& \rho_p = \omega\rho, u = 0, u_p = 0, v_p \rightarrow v \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (2.3)$$

Where ω is the density ratio in the main stream.

Similarly consider the non-dimensional temperature boundary conditions as follows to solve T and T_p

$$\left. \begin{aligned} T = T_w = T_\infty + T_0 \frac{cx^2}{v(1-at)^2} \text{ at } y = 0 \\ T \rightarrow T_\infty, T_p \rightarrow T_\infty \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (2.4)$$

For most of the gases $\tau_p \approx \tau_T$,

$$k_s = k \frac{c_s \mu_s}{c_p \mu} \text{ if } \frac{c_s}{c_p} = \frac{2}{3P_r}$$

Introducing the following non dimensional variables in equation (2.1) to (2.2)

$$\left. \begin{aligned} u = \frac{cx}{1-at} f'(\eta), v = -\sqrt{\frac{cv}{1-at}} f(\eta), \\ u_p = \frac{cx}{1-at} F(\eta), v_p = \sqrt{\frac{cv}{1-at}} G(\eta), \\ \eta = \sqrt{\frac{c}{v(1-at)}} y, \frac{\varphi \rho_s}{\rho} = \frac{\rho_p}{\rho} = \rho_r = H(\eta), \\ P_r = \frac{\mu c_p}{k}, \beta = \frac{1-at}{c\tau_p}, \epsilon = \frac{v_s}{v}, \varphi = \frac{\rho_p}{\rho_s}, \\ A = \frac{a}{c}, E_c = \frac{cv}{c_p T_0}, F_r = \frac{c^2 x}{g(1-at)^2}, \\ G_r = \frac{g\beta^*(T_w - T_\infty)(1-at)^2}{c^2 x}, \gamma = \frac{\rho_s}{\rho}, v = \frac{\mu}{\rho}, \\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \theta_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty} \end{aligned} \right\} \quad (2.5)$$

Where,

$$T - T_\infty = T_0 \frac{cx^2}{v(1-at)^2} \theta, T_p - T_\infty = T_0 \frac{cx^2}{v(1-at)^2} \theta_p$$

The equations (2.1) and (2.2) become

$$H'(\eta) = -\left(H(\eta)F(\eta) + H(\eta)G'(\eta) \right) / \left(A\frac{\eta}{2} + G(\eta) \right) \quad (2.6)$$

$$\begin{aligned} f''''(\eta) + f(\eta)f''(\eta) - (f'(\eta))^2 - A\left(f'(\eta) + \frac{\eta}{2}f''(\eta) \right) + \\ \frac{1}{(1-\varphi)}\beta H(\eta)\left(F(\eta) - f'(\eta) \right) + G_r\theta(\eta) = 0 \end{aligned} \quad (2.7)$$

$$\begin{aligned} A\left(\frac{\eta}{2}F'(\eta) + F(\eta) \right) + (F(\eta))^2 + G(\eta)F'(\eta) - \epsilon F''(\eta) + \\ \beta\left(F(\eta) - f'(\eta) \right) - \frac{1}{F_r}\left(1 - \frac{1}{\gamma} \right) = 0 \end{aligned} \quad (2.8)$$

$$\begin{aligned} \frac{A}{2}\left(\eta G'(\eta) + G(\eta) \right) + G(\eta)G'(\eta) - \epsilon G''(\eta) + \beta\left(f(\eta) + \\ G(\eta) \right) = 0 \end{aligned} \quad (2.9)$$

$$\begin{aligned} \theta'' = Pr(2f'\theta - f\theta') - \frac{2}{3}\frac{\beta}{1-\varphi}H[\theta_p - \theta] - \frac{1}{1-\varphi}PrE_c\beta H\left[F - \\ f' \right]^2 - PrE_c(f'')^2 + \frac{A}{2}Pr\left(\eta\theta'(\eta) + 4\theta(\eta) \right) \end{aligned} \quad (2.10)$$

$$\theta_p'' = \frac{Pr}{\epsilon} \left[\begin{aligned} \frac{A}{2}\left(\theta_p'(\eta)\eta + 4\theta_p(\eta) \right) + 2F(\eta)\theta_p + G(\eta)\theta_p'(\eta) \\ + \beta\left(\theta_p(\eta) - \theta(\eta) \right) \\ + \frac{3}{2}E_cPr\beta\left(f'(\eta) - F(\eta) \right)^2 \\ - \frac{3}{2}\epsilon E_cPr\left(F(\eta)F'(\eta) + (F'(\eta))^2 \right) \end{aligned} \right] \quad (2.11)$$

With boundary conditions

$$\left. \begin{aligned} G'(\eta) = 0, f(\eta) = 0, f'(\eta) = 1, \\ F'(\eta) = 0, \theta(\eta) = 1, \theta_p' = 0 \text{ as } \eta \rightarrow 0 \\ \& f'(\eta) = 0, F(\eta) = 0, G(\eta) = -f(\eta) \\ H(\eta) = \omega, \theta(\eta) = 0, \theta_p = 0 \text{ as } \eta \rightarrow \infty \end{aligned} \right\} \quad (2.12)$$

3. Solution Method:

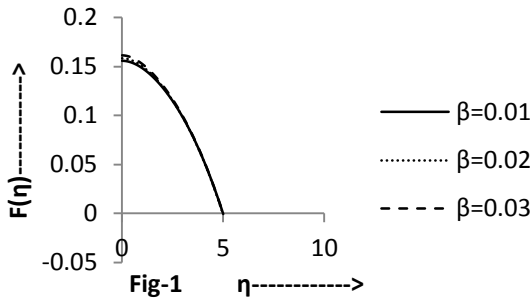
Here in this problem the value of $f''(0), F(0), G(0), H(0), \theta'(0), \theta_p(0)$ are not known but $f'(\infty) = 0, F(\infty) = 0, G(\infty) = -f(\infty), H(\infty) = \omega, \theta(\infty) = 0, \theta_p(\infty) = 0$ are given. We use Shooting method to determine the value of $f''(0), F(0), G(0), H(0), \theta'(0), \theta_p(0)$. We have supplied $f''(0) = \alpha_0$ and $f''(0) = \alpha_1$. The improved value of $f''(0) = \alpha_2$ is determined by utilizing linear interpolation formula. Then the value of $f'(\alpha_2, \infty)$ is determined by using Runge-Kutta method. If $f'(\alpha_2, \infty)$ is equal to $f'(\infty)$ up to a certain decimal accuracy, then α_2 i.e $f''(0)$ is determined, otherwise the above procedure is repeated with $\alpha_0 = \alpha_1$ and $\alpha_1 = \alpha_2$ until a correct α_2 is obtained. The same procedure described above is adopted to determine the correct values of $F(0), G(0), H(0), \theta'(0), \theta_p(0)$.

The solution of the present problem is obtained by numerical computation after finding the infinite value for η . It has been observed from the numerical result that the approximation to $\theta'(0)$ and $f''(0)$ are improved by increasing the infinite value of η which is finally determined as $\eta = 10.0$ with a step length of 0.125 beginning from $\eta = 0$. Depends upon initial guess and number of steps N. FORTRAN-77 is used to find the solutions of problem. The value of $f''(0)$ and $\theta'(0)$ are obtained from numerical computation which is given in table - 1 for different parameters used.

4. Graphical Representations:

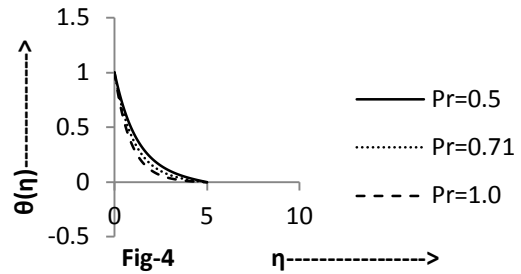
Variation of U_p w.r.t. β

$Ec=1.0, Pr=0.71, Fr=10.0, A=0.2,$
 $Gr=0.01, \phi=0.01, \gamma=1200.0, \epsilon=3.0$



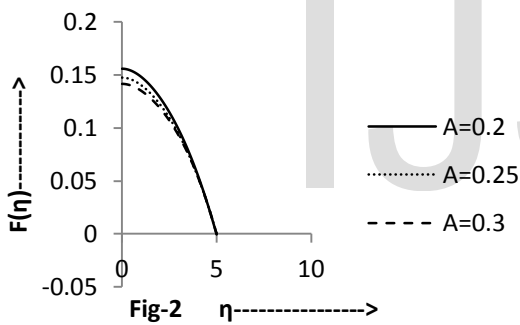
Variation of θ w.r.t. Pr

$A=0.2, Ec=1.0, Fr=10.0, \beta=0.2,$
 $Gr=0.01, \phi=0.01, \gamma=1200.0, \epsilon=3.0$



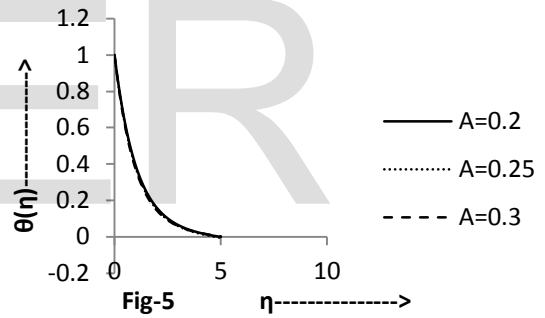
Variation of U_p w.r.t. A

$Ec=1.0, Pr=0.71, Fr=10.0, \beta=0.2,$
 $Gr=0.01, \phi=0.01, \gamma=1200.0, \epsilon=3.0$



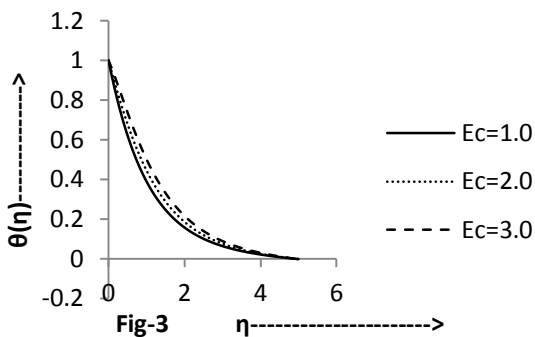
Variation of θ w.r.t. A

$\beta=0.01, Ec=1.0, Fr=10.0, \phi=0.01,$
 $Pr=0.71, Gr=0.01, \gamma=1200.0, \epsilon=3.0$



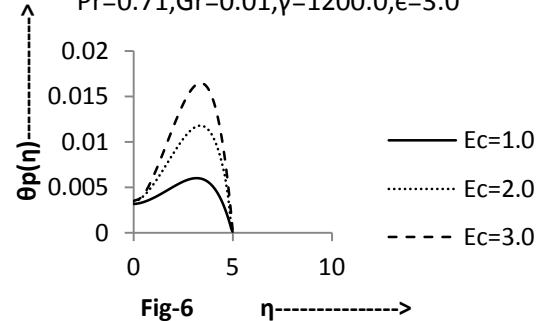
Variation of θ w.r.t. Ec

$A=0.2, Pr=0.71, Fr=10.0, \beta=0.2,$
 $Gr=0.01, \phi=0.01, \gamma=1200.0, \epsilon=3.0$



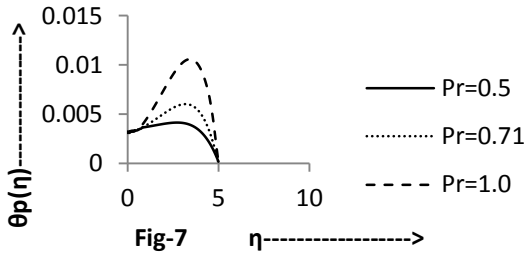
Variation of θ_p w.r.t. Ec

$\beta=0.01, A=0.2, Fr=10.0, \phi=0.01,$
 $Pr=0.71, Gr=0.01, \gamma=1200.0, \epsilon=3.0$



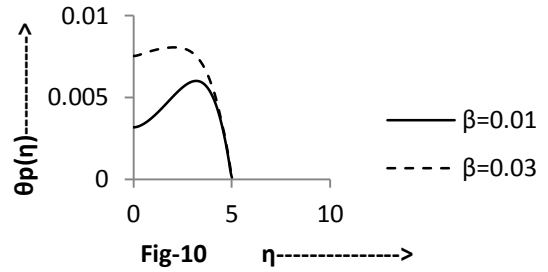
Variation of θ_p w.r.t.Pr

$\beta=0.01, A=0.2, Fr=10.0, \phi=0.01,$
 $Ec=1.0, Gr=0.01, \gamma=1200.0, \epsilon=3.0$



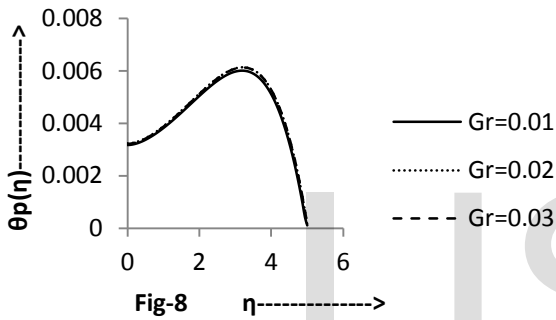
Variation of θ_p w.r.t. β

$Gr=0.01, A=0.2, Fr=10.0, \phi=0.01,$
 $Ec=1.0, Pr=0.71, \gamma=1200.0, \epsilon=3.0$



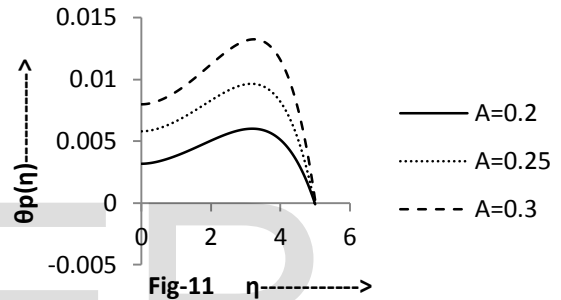
Variation of θ_p w.r.t.Gr

$\beta=0.01, A=0.2, Fr=10.0, \phi=0.01,$
 $Ec=1.0, Pr=0.71, \gamma=1200.0, \epsilon=3.0$



Variation of θ_p w.r.t.A

$\beta=0.01, Gr=0.01, Fr=10.0, \phi=0.01,$
 $Ec=1.0, Pr=0.71, \gamma=1200.0, \epsilon=3.0$



Variation of θ_p w.r.t. ϕ

$\beta=0.01, Gr=0.01, Fr=10.0, \phi=0.01,$
 $Ec=1.0, A=0.2, Pr=0.71, \gamma=1200.0, \epsilon=3.0$

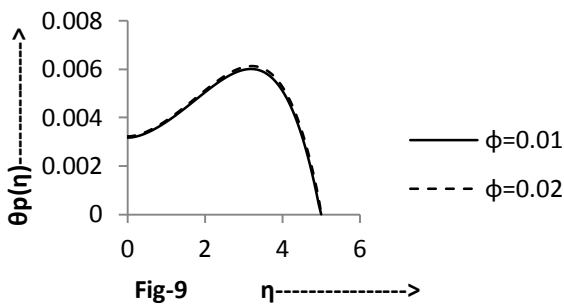


TABLE-1 showing initial values of wall velocity gradient $-f''(0)$ and temperature gradient $-\theta'(0)$

β	A	P_r	E_c	φ	G_r	$-f''(0)$	$-u_p(0)$	$-v_p(0)$	$H(0)$	$-\theta'(0)$	$\theta_p(0)$
0.01	0.23	0.71	0.00	0.00	0.00	1.079240	-	-	-	1.220351	-
0.01	0.2	0.71	1.0	0.01	0.01	1.06509	0.15587	0.72391	0.112037	0.95172	0.003173
			2.0			1.06393	0.155736	0.72406	0.112413	0.70118	0.003446
			3.0			1.06372	0.155619	0.72486	0.112461	0.4491	0.003473
		0.5				1.06407	0.156012	0.72386	0.111769	0.79292	0.003491
0.01	0.2	0.71	1.0	0.01	0.01	1.06509	0.15587	0.72391	0.112037	0.95172	0.003173
		1.0				1.06514	0.155842	0.7247	0.112191	1.13594	0.002766
0.01	0.2	0.71	1.0	0.01	0.01	1.06509	0.15587	0.72391	0.112037	0.95172	0.003173
				0.02		1.06527	0.155453	0.7239	0.112042	0.95163	0.003226
				0.03		1.06537	0.155383	0.72389	0.111939	0.95112	0.004812
0.01	0.2	0.71	1.0	0.01	0.01	1.06509	0.15587	0.72391	0.112037	0.95172	0.003173
					0.02	1.05962	0.156025	0.72396	0.112218	0.9553	0.003233
					0.03	1.05457	0.156055	0.72397	0.112297	0.95821	0.003237
0.01	0.2	0.71	1.0	0.01	0.01	1.06509	0.15587	0.72391	0.112037	0.95172	0.003173
	0.25					1.08135	0.147478	0.66141	0.104831	0.9796	0.005799
	0.3					1.09809	0.141571	0.60725	0.09112	1.00448	0.007985
0.01	0.2	0.71	1.0	0.01	0.01	1.06509	0.15587	0.72391	0.112037	0.95172	0.003173
0.02						1.06585	0.158395	0.72132	0.110086	0.95137	0.004742
0.03						1.06593	0.161402	0.71951	0.110064	0.95124	0.007533

5. Result and Discussion:

The set of non linear ODEs (2.6) to (2.11) with boundary condition (2.12) were solved using well known Runge-Kutta fourth order algorithm with a systematic guessing of $f''(0)$ and $\theta'(0)$ by the shooting technique until the boundary condition at infinity are satisfied. The step size 0.125 is used while obtaining the numerical solution accuracy up to the sixth decimal place i.e. 1×10^{-6} , which is very sufficient for convergence. In this method we choose suitable finite values of $\eta \rightarrow \infty$ which depends on the values of parameter used. The computations were done by the computer language FORTRAN-77. The shear stress (Skin friction coefficient) which is proportional to $f''(0)$ and rate of heat transfer (Nusselt number) which is proportional to $\theta'(0)$ are tabulated in Table-1 for different values of parameter used. It is observed from the table that shear stress and rate of heat transfer decreases on the increase of E_c , whereas it is increasing for increasing values of P_r . The Nusselt number decreases on the increasing of unsteady parameter 'A'. The velocity profiles and temperature profiles also demonstrated graphically.

Fig-1 demonstrates the effect of β which infers that increasing of β increases the particle phase velocity. Fig-2 shows that velocity profile of particle phase is decreasing on the increase of unsteady parameter A.

Fig-3 witnesses that increasing values of E_c , the temperature of fluid phase increases which shows effect on the boundary layer growth.

Fig-4 depicts the effect of P_r on temperature profile of fluid phase. From the figure we observe that, when P_r increase the temperature of fluid phase decreases which states that the viscous boundary layer thickness increases and thermal boundary layer thickness decreases.

Fig-5 explains that the temperature of fluid phase decreases with increasing unsteady parameter 'A'.

Fig-6 illustrates the effect of E_c on temperature profile of particle phase. It is evident that the increasing of E_c increases the temperature.

Fig-7 explains the effect of P_r on particle phase temperature, when P_r is increasing there is increasing of the temperature of particle phase.

Fig-8 depicts the effect of G_r on particle phase temperature profile which indicates that the increasing of G_r has significant effect on particle phase temperature, enhancing G_r , increases the temperature of particle phase.

Fig-9 shows the effect of φ on temperature of particle phase. It is evident that increasing in volume fraction increases the temperature of particle phase which means thermal boundary layer thickness increases.

Fig-10 illustrates the increasing β , increases the temperature of dust phase.

Fig-11 describes that the temperature profile of particle phase increases on the increase of unsteady parameter A.6.

6. Conclusions:

The significant findings of the present study of the unsteady flow of a viscous, incompressible dusty fluid are presented below:

- i. Increasing value of Ec is enhancing the temperature of both fluid phase as well as particle phase which indicates that the heat energy is generated in fluid due to frictional heating.
- ii. The thermal boundary layer thickness of fluid phase decreases on the increase of Pr . But the temperature of particle phase increases on increasing Pr . The temperature decreases at a faster rate for higher values of Pr which implies the rate of cooling is faster in case of higher prandtl number.
- iii. The momentum boundary layer thickness decreases and thermal boundary layer thickness increases on the effect of Gr . If $Gr = 0$ the present study will represent the horizontal stretching sheet.
- iv. Increasing β increases the velocity and temperature profiles of particle phase.
- v. The increasing value of ϕ increases the temperature profile of particle phase.
- vi. The velocity of particle phase decreases on the increase of unsteady parameter A .
- vii. The temperature of fluid phase decreases and temperature particle phase increases on the increase of unsteady parameter A .
- viii. We have investigated the problem using the values $\gamma=1200.0$, $Fr=10.0$, $\epsilon=3.0$.

Nomenclature:

Symbol	Meaning
E_c	eckert number
F_r	froud number
G_r	grashof number
P_r	prandtl number
T_∞	temperature at large distance from the wall.
T_w	wall temperature
T_p	temperature of particle phase.
$U_w(x)$	stretching sheet velocity
c_p	specific heat of fluid
c_s	specific heat of particles
k_s	thermal conductivity of particle
A	unsteady parameter
g	acceleration due to gravity
c	stretching rate
k	thermal conductivity of fluid
l	characterstic length
T	temperature of fluid phase
u, v	velocity component of fluid along x-axis and y-axis

x, y cartesian coordinate
 u_p, v_p velocity component of the particle along x-axis and y-axis

Greek Symbol

ϕ volume fraction
 β fluid – particle interaction parameter
 β^* volumetric coefficient of thermal expansion
 ρ density of the fluid
 ρ_p density of the particle phase
 ρ_s material density
 η similarity variable
 θ fluid phase temperature
 θ_p dust phase temperature
 μ dynamic viscosity of fluid
 ν kinematic viscosity of fluid
 γ ratio of specific heat
 τ relaxation time of particle phase
 τ_T thermal relaxation time i.e. the time required by the dust particle to adjust its temperature relative to the fluid.
 τ_p velocity relaxation time i.e. the time required by the dust particle to adjust its velocity relative to the fluid.

ϵ diffusion parameter
 ω density ratio

Subscripts

w condition at the wall
 ∞ condition at infinity

7. References:

[1] A.Aziz(2009), " A similarity solution for laminar thermal boundary layer over a flat plate with a convective surface boundary condition" , Communications in Nonlinear Science and Numerical Simulation , vol.14,no.4,2009,pp.1064-1068 ,doi:1016/j.cnsns.2008.05.003.

[2] B.J.Gireesha ,H.J.Lokesh ,G.K.Ramesh and C.S.Bagewadi(2011), "Boundary Layer flow and Heat Transfer of a Dusty fluid over a stretching vertical surface" , Applied Mathematics,2011,2,475-481(<http://www.SciRP.org/Journal/am>) ,Scientific Research.

[3] B.J.Gireesha, A.J. Chamakha, S.Manjunatha and C.S.Bagewadi(2013), " Mixed convective flow of a dusty fluid over a vertical stretching sheet with non uniform heat source/sink and radiation" ; International Journal of Numerical Methods for Heat and Fluid flow,vol.23.No.4,pp.598-612,2013

[4] B.J.Gireesha,G.S.Roopaa and C.S.Bagewadi (2011), "Boundary Layer flow of an unsteady Dusty fluid and Heat Transfer over a stretching surface with non uniform heat source/sink " , Applied Mathematics,2011

- ,3,726-735. (<http://www.SciRP.org/Journal/am>), Scientific Research.
- [5] B.J.Gireesha,S.Manjunatha and C.S.Bagewadi(2014), "Effect of Radiation on Boundary Layer Flow and Heat Transfer over a stretching sheet in the presence of a free stream velocity";Journal of Applied fluid Mechanics,Vol.7,No.1,pp.15-24,2014
- [6] C.H.Chen(1998) , "Laminar Mixed convection Adjacent to vertical continuity stretching sheet, "Heat and Mass Transfer,vol.33,no.5-6,1998,pp.471-476.
- [7] Chamakha,A.J.(2004).Unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption.Int.J.Engng.Sci.42,217-230.
- [8] G.K.Ramesh , B.J.Gireesh and C.S.Bagewadi(2012), "Heat Transfer in M.H.D Dusty Boundary Layer flow of over an inclined stretching surface with non uniform heat source/sink " ,Hindawi Publishing Corporation , Advances in Mathematical Physics,volume-2012,Article ID 657805,13 pages.
- [9] Grubka L.J. and Bobba K.M(1985), "Heat Transfer characteristics of a continuous stretching surface with variable temperature" , Int.J.Heat and Mass Transfer , vol.107,pp.248-250 , 1985.
- [10] H.I.Anderson,K.H.Bech and B.S.Dandapat(1992), "MHD flow of a power law fluid over a stretching sheet" , Int.J.of Nonlinear Mechanics,vol.27,no.6,pp.929-936,1992.
- [11] H.Schlichting(1968), "Boundary Layer Theory" , McGraw-Hill , New York,1968.
- [12] K.M.Chakrabarti(1974) , "Note on Boundary Layer in a dusty gas" ,AIAA Journal , vol.12.no.8,pp.1136-1137,1974.
- [13] L.J.Crane(1970), "Flow past a stretching plate" , Zeitschrift fur Angewandte Mathematik und physic ZAMP,VOL.2,NO.4,PP.645-647.1970.
- [14] M.Das, B.K.Mahanta, R.Nandkeyolyar, B.K.Mandal and K.Saurabh(2014), Unsteady Hydromagnetic flow of heat absorbing dusty fluid past a permeable vertical plate with ramped temperature. Vol.7, No.3, pp485-492, 2014.
- [15] N.Datta and S.K.Mishra(1982) , "Boundary layer flow of a dusty fluid over a semi-infinite flat plate" ,Acta Mechanica,vol.42.no1-2,1982,pp71-83.doi:10.1007/BF01176514.
- [16] Nandkeolyar, R.and M.Das(2013)Unsteady MHD flow of a heat absorbing dusty fluid past a flat plate with ramped wall temperature. Afr. Mat. Article in Press.
- [17] Saffman ,P.G.(1962), "On the Stability of Laminar flow of a dusty gas" , Journal of Fluid Mechanics,13,120-128.
- [18] Sakiadis B.C (1961), "Boundary Layer behavior on continuous solid surface ; boundary layer equation for two dimensional and axisymmetric flow" A.I.Ch.E.J.,Vol.7,pp 26-28.
- [19] Sharidan S. , Mahmood J. and Pop I. (2008), "Similarity solutions for the unsteady boundary layer flow and Heat Transfer due to a stretching sheet" , Int.J. of Appl.Mechanics and Engeenering,vol.11,No.3,pp 647-654.
- [20] Tsou,F.K,E.M. Sparrow , R.J. Glodstein (1967), "Flow and Heat Transfer in the boundary layer on a continous moving surface" , Int .J. Heat and Masstransfer,10,219-235.
- [21] Vajravelu , K. and Nayfeh(1992) , J. , "Hydromagnetic flow of a dusty fluid over a stretching sheet" , Int.J. of nonlinear Mechanics,vol.27,No.6,pp.937-945.

Aswin Kumar Rauta was born in Khallangi of district Ganjam, Odisha, India in 1981.He obtained the M.Sc. and M.Phil. degree in Mathematics from Berhampur university, Berhampur , Odisha, India. He has qualified NET in 2009 conducted by CSIR-UGC, government of India. He joined as a lecturer in Mathematics in the Department of Mathematics, S.K.C.G.College, Paralakhemundi ,Odisha,India in 2011 and is continuing his research work since 2009 and work till now. His field of interest covers the areas of application of boundary layer,heat/mass transefer and dusty fluid flows.**Dr.Saroj Kumar Mishra** was born in Narsinghpur of Cuttuck district,Odisha,India on 30th june 1952.He received his M.Sc. degree in Mathematics (1976) and Ph.D in Mathematics in 1982 on the research topic "Dynamics of two phase flow" from IIT Kharagpur, India . Currently he is working as Adjunct Professor of Mathematics at Centre for Fluid Dynamics Research, CUTM, Paralakhemundi, Odisha, India. He has authored and coauthored 50 research papers published in national and international journal of repute. He has completed one Major Research project and one Minor Research project sponsored by UGC, New Delhi, India. He has attended/presented the papers in national, international conferences. He is a member of several bodies like Indian Science Congress Association, Indian Mathematical Society, ISTAM, OMS, and BHUMS etc. His research interest includes the area of fluid dynamics, dynamics of dusty fluid particularly, in boundary layer flows, heat transfer, MHD, FHD and flow through porous media. His research interest also covers the nano fluid problems, existence and stability of problems and other related matters. Eight students have already awarded Ph.D degree under his guidance and another six students are working under his supervision.